MA114 Summer 2018
Worksheet 7 – Sequences – 6/19/18

1. (a) What does it mean to say that $\lim_{x\to a} f(x) = L$? Does this differ from $\lim_{n\to\infty} f(n) = L$? Why or why not?
   (b) What does it mean for a sequence to converge?
   (c) Sequences can diverge in different ways. Describe at least two ways that a sequence can diverge.
   (d) Give two examples of sequences that converge to 0 and two examples of sequences that converge to a given number $L \neq 0$.

2. Write the first five terms of the sequences with the following general terms.
   (a) $\dfrac{n!}{2^n}$
   (b) $\dfrac{n}{n+1}$
   (c) $(-1)^{n+1}$
   (d) $\{a_n\}_{n=1}^{\infty}$, where $a_n = \dfrac{3}{n}$.

3. Find a formula for the n^{th} term of the sequence.
   (a) $\left\{\dfrac{1}{1}, -\dfrac{1}{8}, \dfrac{1}{27}, -\dfrac{1}{64}, \ldots\right\}$
   (b) $\left\{1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16} \ldots\right\}$
   (c) $\{1, 0, 1, 0, 1, 0 \ldots\}$

4. Suppose that a sequence is bounded above and below. Does it have to converge? If not, give a counterexample.

5. Remember that the limit laws for sequences are the same as the limit laws for functions. Suppose you have sequences $\{a_n\}, \{b_n\}$, and $\{c_n\}$ with $a_n \to 15, b_n \to 0$, and $c_n \to 1$. Use the limit laws to answer the following questions.
   (a) Does the sequence $\left\{\dfrac{a_n \cdot c_n}{b_n + 1}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?
   (b) Does the sequence $\left\{\dfrac{a_n + 3 \cdot c_n}{2 \cdot b_n + 2}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?

6. Write out the first five terms of
   (a) $a_0 = 0, a_1 = 1$, and $a_{n+1} = 3a_{n-1} + a_n^2$.
   (b) $a_1 = 6, a_{n+1} = \dfrac{a_n}{n}$.
   (c) $a_1 = 2, a_{n+1} = \dfrac{a_n}{a_n + 1}$.
   (d) $a_1 = 2, a_2 = 1$, and $a_{n+1} = a_n - a_{n-1}$.

7. Assuming that the limit of the sequence exists, find the limit of the recursive sequence given by $a_1 = 1, a_n = \dfrac{1}{2}(a_{n-1} + \frac{2}{a_{n-1}})$.